Letting $\eta(t) = \eta_0 e^i \omega e^t$ and obtaining the Laplace transform of Eq. (A21) with zero initial conditions, we obtain the heave transfer function

$$z(s)/\eta = (b_1 s + b_0)/a_3 s^3 + a_2 s^2 + a_1 s + a_0) \quad \text{(A22)}$$

where

$$a_3 = 1 \qquad b_1 = -\frac{1}{M_c}\left(\frac{K_{\theta D}}{B} + C_B A_B\right) I_1$$

$$a_2 = d_3/M_c - C_B R_1 \qquad b_0 = \frac{C_B I_1}{M_C}\left(R_1 \frac{K_{\theta D}}{B} - G\right)$$

$$a_1 = \frac{K_{\theta D} L + C_B (A_B{}^2 - d_z R_1)}{M_c}$$

$$a_0 = \frac{C_B (A_B G - K_{\theta D} L R_1)}{M_c}$$

The cubic polynominal equation

$$P(s) = a_3 s^3 + a_2 s^2 + a_1 s + a_0 = 0 \quad \text{(A23)}$$

is the characteristic equation of the system and Eq. (A22) is the heave transfer function $z(s)/\eta$.

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Systems Identification: Application to Underwater Vehicle Dynamics

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A deterministic method of systems identification is presented in a generalized form applicable to numerous lumped-parameter mathematical models. The minimization of a measured error between experimental and calculated time records results in the optimum or best-fit model parameters and constitutes the basic identification process. The iterative search method suggested for optimization does not necessitate a good initial parameter estimate. The procedure is exemplified by an application to the governing equations of motion for an underwater vehicle. The hydrodynamic coefficients form the set of parameters for optimization. Coefficient retrieval using a theoretically generated record and the prediction of coefficient values from experimental records is investigated. The problem of numerical sensitivity is sighted. Although coefficients are determined for excellent agreement in the trajectories, a refined approach is suggested to reduce the number of parameters and improve the efficiency of search.

Nomenclature

x, y, z = body axes of vehicle
x_g, z_g = distances from CB to CG
$\delta_e(t)$ = elevator input
U = magnitude of translational velocity
α = angle of attack
w = transverse component of velocity
θ = angle of pitch
m, W = mass and weight of vehicle, respectively
l = length of vehicle
B = buoyant force
ρ = mass density of fluid medium
I_y = moment of inertia about y-axis
Z', M' = hydrodynamic coefficients with respect to forces and moments
Z_o = depth of vehicle at CB
$\bar{x}$ = state vector
$\bar{x}_c$ = computed solution to motion equations
$\bar{x}_e$ = experimental record of state
$\bar{x}_o$ = initial-value vector
t = independent time variable
$\bar{\eta}$ = (10 × 1) vector of hydrodynamic coefficients
ϵ = objective error function
{ } = denotes a column vector
[ ] = denotes a row vector
$\bar{\eta}^0$ = vector of initial estimates
$\bar{\eta}^*$ = denotes the value of $\bar{\eta}$ for a minimal ϵ
ϕ = (4 × 10) null matrix

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Introduction

THE analytical prediction of the dynamic response of an underwater vehicle is a function of a set of parameters commonly referred to as hydrodynamic coefficients. For a given body configuration, reasonable numerical estimation of these parameters is required for any accurate simulation of vehicle performance. Water tunnel tests, theoretical predictions, and trial-and-error adjustments of the coefficients of motion to obtain an observed numerical simulation of experimental records constitute the existing techniques of determining the hydrodynamic properties of an underwater vehicle. These methods are costly, time consuming, and do not provide any mathematically defined optimum. The present study is motivated by the need to develop a direct and concise method of evaluating the hydrodynamic coefficients.

In general, this study is formulated as the identification of model parameters through the direct minimization of an objective function defined by the sum of squared errors between calculated and experimental records of the state variables. Similar procedures have been employed successfully in Refs. 1–3 where the dynamic motion of surface ships and aircraft is characterized by the identification of optimal parameters. In the present study the response of an underwater vehicle having kinematic symmetry with respect to the pitch plane is described. Thus, the equations of motion become simplified by the conditions of zero roll and yaw. This simplification provides a set of equations which are convenient for the purpose of testing the feasibility of optimum parameter identification. However, sufficient generality is maintained for immediate application of results, and the extension of the technique to a generalized six-degree-of-freedom model can be achieved without extreme modifications.

The results indicate that the fundamental questions regarding the uniqueness and identifiability of a minimal parameter set need to be answered. An in-depth theoretical study of system characteristics is recommended to address these questions and supplement the present numerical study.

Pitch-Plane Motion Equations

The motion of an underwater vehicle having geometric and mass symmetry with respect to the vertical xz-plane is illustrated in Fig. 1. The xyz body coordinate system is placed with its origin at the center of buoyancy, and the x, y, z axes are the principal axes of inertia for the body. The simplified state of plane motion is considered as the vehicle responds to a force field with xz-plane symmetry. In the pitch plane the elevator deflection $\delta_e(t)$ is the primary control function. The following conditions are assumed to be valid at any instant in the propelled motion of the vehicle: 1) The trust vector acts along the x-axis. 2) The magnitude of the translational velocity U remains constant. 3) The angle of attack remains small, allowing linearization with respect to α.

These assumptions are realistic approximations in the mathematical representation of submerged vehicle motion exhibiting dynamic stability. With the implications of the above assumptions, the governing equations of pitch-plane motion are found to be[4]

$$m\dot{w} - mx_g\ddot{\theta} - mU\dot{\theta} - mz_g\dot{\theta}^2 - (W-B)\cos\theta = \frac{\rho}{2}l^3Z'_{\dot{w}}\dot{w} + \frac{\rho}{2}l^2Z'_wUw + \frac{\rho}{2}l^3Z'_qU\dot{\theta} + \frac{\rho}{2}l^4Z'_{\dot{q}}\ddot{\theta} + \frac{\rho}{2}l^2U^2Z'_{\delta e}\delta_e \tag{1}$$

for the equation of dynamic forces and

$$I_y\ddot{\theta} - mx_g\dot{w} + mx_gU\dot{\theta} + mz_gw\dot{\theta} + Wz_g\sin\theta + Wx_g\cos\theta = \frac{\rho}{2}l^4M'_{\dot{w}}\dot{w} + \frac{\rho}{2}l^3M'_wUw + \frac{\rho}{2}l^4M'_qU\dot{\theta} + \frac{\rho}{2}l^5M'_{\dot{q}}\ddot{\theta} + \frac{\rho}{2}l^3U^2M'_{\delta e}\delta_e \tag{2}$$

for the equation of dynamic moments. The subscripted coefficients Z' and M' represent the hydrodynamic properties of the body. The notation ($\dot{}$) indicates differentiation with respect to the independent time variable t. It is noted that the relationship

$$u = U\cos\alpha \cong U$$

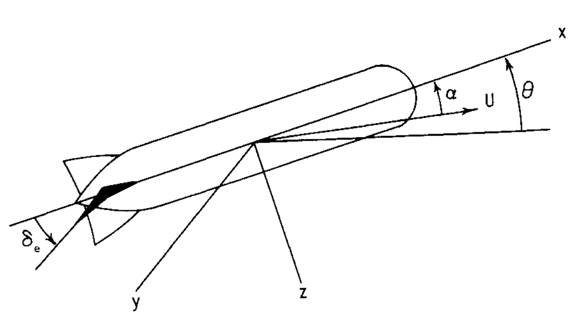

Fig. 1 Motion of an underwater vehicle having geometric and mass symmetry with respect to the vertical xz-plane.

in the presence of small angles of attack is utilized to determine the longitudinal velocity u. The vertical position of the center of buoyancy is traced by the equation

$$\dot{z}_0 = -U\sin\theta + w\cos\theta \tag{3}$$

Thus, the depth and orientation of the vehicle is completely defined in terms of the dependent variables θ and w.

The division of Eq. (1) by ρl^4 and Eq. (2) by ρl^5 yields

$$[\tilde{m} - 1/2Z'_{\dot{w}}]\dot{\chi} - [\tilde{m}\tilde{x}_g + 1/2Z'_{\dot{q}}]\ddot{\theta} = [\tilde{m} + 1/2Z'_q]\Gamma\dot{\theta} + 1/2Z'_w\Gamma\chi + \tilde{m}\tilde{z}_g\dot{\theta}^2 + 1/2\Gamma^2Z_{\delta e}\delta_e + (\tilde{W} - \tilde{B})\cos\theta \tag{4a}$$

$$[\tilde{I}_y - 1/2M'_{\dot{q}}]\ddot{\theta} - [\tilde{m}\tilde{x}_g + 1/2M'_{\dot{w}}]\dot{\chi} = [-\tilde{m}\tilde{x}_g + 1/2M'_q]\Gamma\dot{\theta} + 1/2M'_w\Gamma\chi - \tilde{m}\tilde{z}_g\chi\dot{\theta} + 1/2\Gamma^2M'_{\delta e}\delta_e - \tilde{W}\tilde{z}_g\sin\theta - \tilde{W}\tilde{x}_g\cos\theta \tag{4b}$$

and

$$\dot{\xi}_0 = -\Gamma\sin\theta + \chi\cos\theta \tag{4c}$$

where the parameters $\tilde{m} = m/\rho l^3$, $\tilde{I}_y = I_y/\rho l^5$, $\tilde{W} = W/\rho l^4$, $\tilde{B}/\rho l^4$, $\tilde{x}_g = x_g/l$, $\tilde{z}_g = z_g/l$, and $\Gamma = U/l$ are defined in terms of the inherent vehicle properties and the mass density of the surrounding fluid. The redefined variables are $x = w/l$, $\xi_0 = z_0/l$.

For a given form of the control function input δ_e, the integration of Eq. (4) results in a theoretical simulation of vehicle performance. The hydrodynamic coefficients Z' and M' in Eq. (4) are lumped (i.e., spatially invariant) parameters representing the hydrodynamic forces and moments imparted to the body by a three-dimensional flowfield. The establishment of realistic numerical values for the force coefficients Z' and moment coefficients M' which are characteristic of a given vehicle configuration presents formidable difficulties. The following procedure provides an effective method of identifying the hydrodynamic coefficients by the direct minimization of a functional error between a theoretically computed response and an actual experimental record of state variable histories.

Identification of System Parameters

For the present approach it is assumed that the mass/volume distribution of the vehicle is a known or a directly measurable quantity, and the parametric uncertainty in the system equations lies totally in the hydrodynamic coefficients. The technique outlined applies to the identification of parameters in a set of generalized dynamic system equations.

The governing equations of motion (4) may be written in vectorial notation as four first-order equations

$$d\bar{x}/dt = \bar{f}(\bar{x},\delta_e;\bar{\eta});\ t > 0 \tag{5a}$$

where

$$\bar{x} = \begin{Bmatrix} \chi \\ \theta \\ \dot{\theta} \\ \xi_0 \end{Bmatrix} = \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix}$$

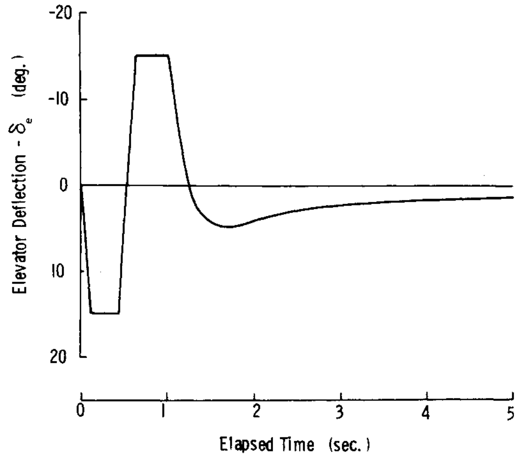

Fig. 2 Theoretical input δ_e in coefficient retrieval.

and $\bar{f}$ is the appropriately defined $(\bar{4} \times 1)$ vector function. The initial conditions are taken to be known quantities

$$\bar{x}(0) = \bar{x}_0 \tag{5b}$$

at the time $t = 0$.

Note that in some system identification studies[1-3] the initial condition $\bar{x}_o$ are assumed to be unknown and constitute part of the set of parameters to be identified. The identification technique described here is capable of encompassing this case; however, for this study the values of $\bar{x}_o$ are assumed to be known precisely. The appearance of the secondary arguments $\bar{\eta}$ in Eq. (5a) with

$$\bar{\eta}^T = [Z'_{\dot{w}} Z'_{\dot{q}} M'_{\dot{w}} M'_{\dot{q}} Z'_w Z'_q M'_w M'_q Z'_{\delta e} M'_{\delta e}]$$

exhibits the parametric dependence of the system (superscript T indicates transpose). It is now supposed that for a given input $\delta_e(t)$ a solution to the initial-value problem, Eq. (5), is obtained on the interval $[0, t_N]$. This theoretical or computed solution is denoted by

$$\bar{x}_c = \bar{x}_c(t;\bar{\eta}); \quad 0 \leq t \leq t_N \tag{7}$$

where $\bar{\eta}$ is the $(\bar{1}\bar{0} \times 1)$ vector of hydrodynamic coefficients. Let the available experimental records of state variable histories by symbolized by

$$\bar{x}_e = \bar{x}_e(t); \quad 0 \leq t \leq t_N \tag{6}$$

The absence of data may occur in the form of a null element of $\bar{x}_e$. It is noted that the expressions (6) and (7) may represent discrete functions of time. Thus, numerical solutions for $\bar{x}_c$ and discrete data points for $\bar{x}_e$ are allowed.

In terms of the basic elements of analysis presented above, the express purpose of this investigation is to find the $\bar{\eta}$ which results in a minimal measure of error between $\bar{x}_c$ and $\bar{x}_e$. The objective function defined as

$$\epsilon = \sum_{i=1}^{N} \bar{\varphi}_i^T(\bar{\eta}) A \varphi_i(\bar{\eta}) \tag{8}$$

with

$$\bar{\varphi}_i(\bar{\eta}) = \bar{x}_c(t_i;\bar{\eta}) - \bar{x}_e(t_i)$$

is utilized to measure the difference between computed and experimental records of the state variables at each of the N points in time t_i. The diagonal matrix A is a constant $(\bar{4} \times 4)$ weighting matrix and the positive definite nature of the quadratic form Eq. (8) follows directly from the definition of ϵ. It is now apparent that the optimal parameter vector $\bar{\eta}^*$ is determined by the necessary conditions

$$\{\partial\epsilon/\partial\bar{\eta}\}_{\bar{\eta}=\bar{\eta}^*} = \bar{0} \tag{9}$$

for a minimum of the error function in expression (8). Presently, the optimization of $\bar{\eta}$ is expressed as the problem of generating a solution $\bar{x}_c(t;\bar{\eta}^*)$ to the governing equations (5) which results in the null gradient vector in Eq. (9). Differentiation of expression (8) with respect to $\bar{\eta}$ yields

$$\{\partial\epsilon/\partial\bar{\eta}\} = 2\sum_{i=1}^{N}(J_i)^T A \bar{\varphi}_i \tag{10}$$

where

$$(J_i) = \partial\bar{\varphi}_i/\partial\bar{\eta} = \partial\bar{x}_c(t_i;\bar{\eta})/\partial\bar{\eta}$$

is the $(\bar{4} \times 10)$ Jacobian matrix evaluated at t_i. The nonlinear form of the vector function $\bar{f}$ in the governing Eqs. (5) prohibits closed-form solution in terms of the parameters $\bar{\eta}$. Consequently, the gradient vector in Eq. (10) cannot be evaluated explicitly, and a supplementary method must be devised for the purpose of determining the Jacobian matrix (J_i). Differentiation of the initial-value Eqs. (5) with respect to $\bar{\eta}$ results in the variational problem

$$d/dt(\partial\bar{x}/\partial\bar{\eta}) = (\partial\bar{f}/\partial\bar{x})(\partial\bar{x}/\partial\bar{\eta}) + \partial\bar{f}/\partial\bar{\eta} \tag{11a}$$

$$(\partial\bar{x}/\partial\bar{\eta})_{t=0} = \phi \tag{11b}$$

which is composed of 40 first-order equations. For a given set of parameters $\bar{\eta}$ the simultaneous integration of Eqs. (5) and Eqs. (11) from $t = 0$ to $t = t_N$ generates the functions

$$\bar{x}_c(t;\bar{\eta}) \tag{12a}$$

$$\partial\bar{x}_c(t;\bar{\eta})/\partial\bar{\eta} \tag{12b}$$

on the interval $[0, t_N]$. Substitution of functions (12) into Eq. (10) established the components of the gradient vector at the current value of $\bar{\eta}$.

With the gradient vector determined for a designated vector $\bar{\eta}$ which lies in some neighborhood of $\bar{\eta}^*$ a correction $\Delta\bar{\eta}$ is desired such that

$$\bar{\eta} + \Delta\bar{\eta} \rightarrow \bar{\eta}^*$$

i.e., the corrected value of $\bar{\eta}$ approaches $\bar{\eta}^*$. In the following, a modified Gauss-Levenberg method[5] is formulated and implemented for the purpose of obtaining the successive corrections $\Delta\bar{\eta}$. Assuming negligible changes of the Jacobian matrix (J_i) with respect to $\bar{\eta}$, the resultant truncated Taylor series of the gradient vector about $\bar{\eta}$

$$\bar{0} = \left\{\frac{\partial\epsilon}{\partial\bar{\eta}}\right\}_{\bar{\eta}} + \left[2\sum_{i=1}^{N}(J_i)^T A (J_i)\right]_{\bar{\eta}} \Delta\bar{\eta} \tag{13}$$

provides an equation for the correction $\Delta\bar{\eta}$. Since the $\varphi_i(\bar{\eta})$ are not truly linear functions of η, the solution to Eq. (13) for $\Delta\bar{\eta}$ must be viewed as an approximate linear extrapolation. An alternate search method is described by the equation

$$\bar{0} = \left\{\frac{\partial\epsilon}{\partial\bar{\eta}}\right\}_{\bar{\eta}} + \lambda\Delta\bar{\eta} \tag{14}$$

where the correction descends toward the minimum in the direction of the gradient. The superposition of the steps $\Delta\bar{\eta}$ predicted by Eqs. (13) and (14) is achieved in the form

$$\bar{0} = \mu\{\partial\epsilon/\partial\bar{\eta}\}_{\bar{\eta}} + \left[2\sum_{i=1}^{N}(J_i)^T A (J_i) + \lambda I\right]_{\bar{\eta}} \Delta\bar{\eta} \tag{15}$$

where μ scales the magnitude of the correction and λ controls the direction of descent. Since the solution $\Delta\bar{\eta}$ to Eq. (15) is not an exact step, the optimum set $\bar{\eta}^*$ cannot be attained in one move. However, successive steps $\Delta\bar{\eta}$ can be applied iteratively in the form of analytical continuation. With appropriate choices of μ and λ convergence to $\bar{\eta}^*$ from a given initial estimate may be obtained.

In summary, the optimization procedure outlined above involves the successive integration of Eq. (5) and (11) in

Table 1 Results of numerical optimization-coefficient retrieval

Coefficients	Initial set η^0	Optimal set $\bar{\eta}^*$	True set $\bar{\eta}_T$
$Z_{\dot{w}}'$	−0.04	−0.03513	−0.03091
$Z_{\dot{q}}'$	−0.0015	−0.000952	−0.001132
$M_{\dot{w}}'$	0.0	0.000858	−0.001132
$M_{\dot{q}}'$	−0.0003	−0.002105	−0.002019
Z_w'	−0.03	−0.0319	−0.0288
Z_q'	−0.013	−0.00826	−0.01014
M_w'	0.009	0.01473	0.01329
M_q'	−0.01	−0.007463	−0.006589
$Z'\delta_e$	−0.007	−0.00629	−0.00602
$M'\delta_e$	−0.001	−0.00207	−0.00220
Error ϵ	1.3×10^2	5.4×10^{-10}	3.4×10^{-10} (truncated data)

conjunction with subsequent correction of $\bar{\eta}$ according to the prediction of Eq. (15). As a result, a numerical sequence of values for $\bar{\eta}$ is generated which converges to $\bar{\eta}^*$. Within the limits of numerical computation an approximation of $\bar{\eta}^*$ may be obtained in terms of a prescribed convergence criterion.

Numerical Evaluation

The theoretical analysis presented above is suitable for the direct employment of a numerical integration technique. Thus, by integrating the initial-value problems, Eqs. (5) and (11), simultaneously with a fourth-order Runge-Kutta-Gill method and performing the successive corrections $\Delta\bar{\eta}$ predicted by a Gauss-Jordan reduction of Eqs. (15), a computed $\bar{\eta}^*$ and the corresponding numerical solution $\tilde{X}_c(t;\bar{\eta}^*)$ may be obtained.

The values

$$\tilde{m} = 0.01489 \qquad \tilde{I}_y = 0.00104$$

$$\tilde{x}_g = -0.02096 \qquad \tilde{z}_g = 0.00206$$

$$\tilde{W} = 0.04089 \text{ sec}^{-2} \qquad \tilde{B} = 0.03998 \text{ sec}^{-2}$$

$$\Gamma = 4.1 \text{ sec}^{-1}$$

are utilized as a representative set of known torpedo parameters. For these particular vehicle characteristics the feasibility of the proposed numerical optimization is examined by retrieving coefficients from theoretically generated data and extracting coefficients from actual experimental data.

In the numerical integration a step-size of $\Delta h = 0.05$ sec was used for computing a state variable record length of five seconds. The residuals in pitch θ and dimensionless depth ξ_0 were used in constructing the objective function ϵ with a sampling interval $\Delta t_i = t_{i+1} - t_i = 0.2$ sec ($N = 25$). Equivalent contributions to the error measure ϵ by unit residuals in degrees of pitch and feet of depth were provided by the appropriate choice of the nonzero elements of the weighting matrix A ($a_{22} = 25$, $a_{44} = 1$). The correction factors used in determining $\Delta\bar{\eta}$

$$\lambda = \max_{1 \leq k \leq 10} \left| \frac{\partial \epsilon}{\partial \eta_k} \right|$$

$$\mu = \begin{cases} 0.5 & \lambda > 1 \\ 1/(1+\lambda) & \lambda \leq 1 \end{cases}$$

were found to yield convergence with a monotone decreasing ϵ function in all test cases. Convergence with $\lambda = 0$ was not achieved regularly due to severe linear extrapolations from poor initial guesses. The suggested values of λ and μ allowed poor initial estimates, say $\bar{\eta}^0$, of $\bar{\eta}^*$ without a resulting divergence of the sequential corrections. It is noted that this is achieved by retarding the initial rate of descent.

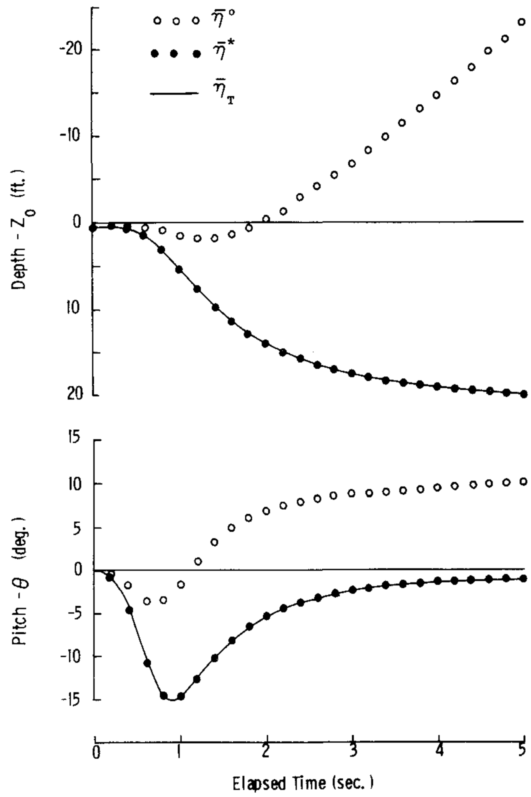

Fig. 3 Simulated trajectories in coefficient retrieval.

Coefficient Retrieval

The ability to establish the coefficients attached to a theoretical data record demonstrates feasibility and establishes confidence in the identification procedure. Figure 2 illustrates the elevator control $\delta_e(t)$ exhibited by an autopilot model during a digital simulation of vehicle response to a relative depth command of twenty feet. Figure 3 describes the response of the vehicle generated by integrating Eqs. (5) with the illustrated input $\delta_e(t)$ and a so-called true set of hydrodynamic coefficients $\bar{\eta}_T$. These simulated traces of pitch and depth constitute the pseudo-experimental record used in a numerical test of the ability to retrieve the true coefficients inherent in the data. As listed in Table 1, the search was initiated at the arbitrarily chosen estimate $\bar{\eta}^0$ and convergence to $\bar{\eta}^*$ was achieved using four decimal-place data with eighty-four iterations. The related trajectories are shown in Fig. 3. This example typifies the results obtained in the identification of hydrodynamic coefficients. Although apparent deviations are shown between the returned coefficients $\bar{\eta}^*$ and the true coefficients $\bar{\eta}_T$, the related trajectories are nearly identical in terms of the four decimal-place data. Thus, these apparent deviations are insignificant in the theoretical prediction of vehicle response. A possible explanation for this behavior may be in the choice of residual weighting factors (the elements of A) in conjunction with an insensitivity of ϵ with respect to $\bar{\eta}$ near the optimum $\bar{\eta}^*$. It should be mentioned that an obvious insensitivity to the inertia coupling coefficients Z'_q and M'_w is exhibited by relatively large variations in the values of these coefficients with little effect upon ϵ in the final stages of the search for $\bar{\eta}^*$. Since a detailed and rigorous sensitivity analysis is not the primary objective of the present study, this area is left open for future investigation. The results obtained and illustrated above demonstrate the feasibility of numerical optimization with ideal data. The exemplified level of precision in determining $\bar{\eta}^*$ is more than adequate for the

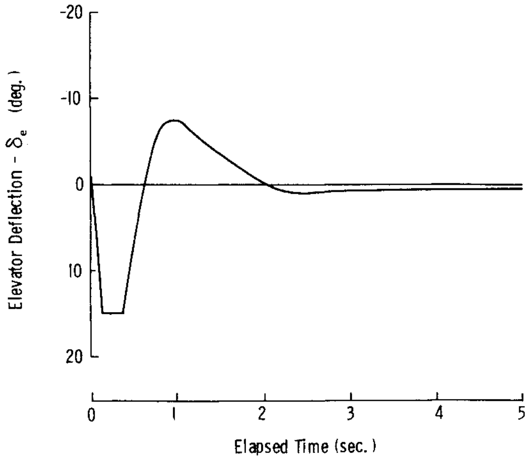

Fig. 4 Experimental input δ_e in coefficient identification.

purpose of model simulation studies. The identification of hydrodynamic coefficients inherent in actual experimental test data remains as a final measure of practical applicability.

Coefficient Identification

The elevator control $\delta_e(t)$ exhibited in an actual vehicle test run is shown in Fig. 4. The coinciding response in terms of pitch and depth is traced in Fig. 5. The iterative search was initiated at $\bar{\eta}^\circ$ and terminated with convergence using the criterion

$$\max_{1 \leq k \leq 10} |\partial\epsilon/\partial\eta_k| < 10^{-3}$$

The results are listed and illustrated in Table 2 and Fig. 5, respectively. The simulation of pitch and depth with $\bar{\eta}$

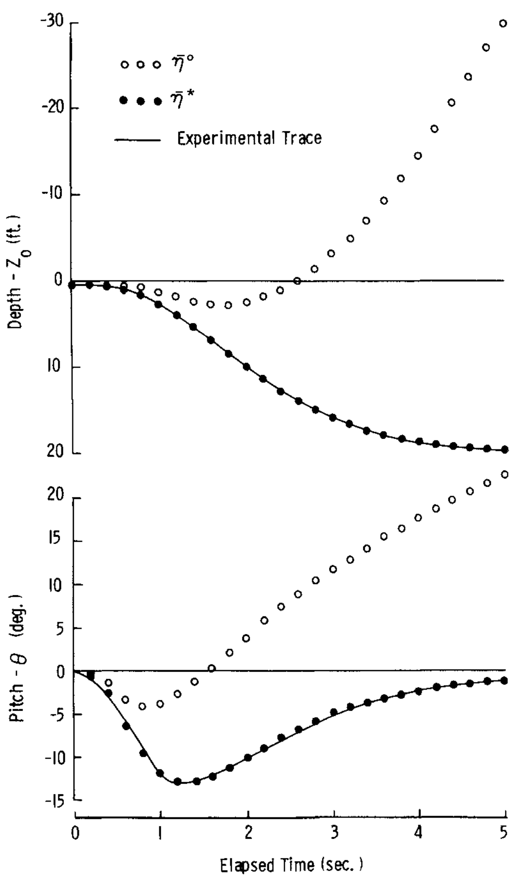

Fig. 5 Experimental and simulated trajectories in coefficient identification.

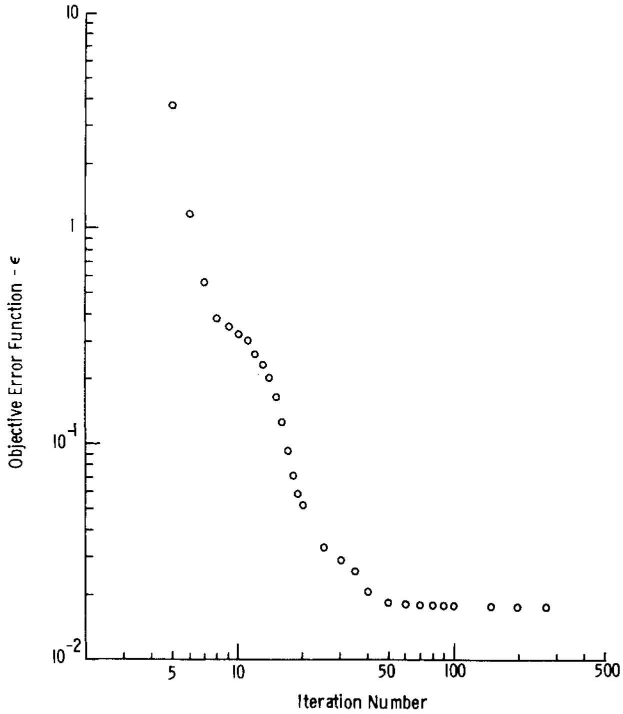

Fig. 6 Objective error ϵ vs number of iterations.

$= \bar{\eta}^*$ in the governing equations of motion is seen to be in excellent agreement with the corresponding experimental records. The rate of convergence is represented in Fig. 6 where the objective error ϵ is shown as a function of the iteration number. It is noted that ϵ is monotone decreasing. Although convergence is achieved, the total number of required iterations is large. Of the total number of iterations, approximately two-hundred are required to reduce the magnitude of the objective error by $\Delta\epsilon = 4 \times 10^{-4}$. The corresponding changes in the elements of the coefficient vector $\bar{\eta}$ are relatively large, as shown by example in Table 3. This behavior is characteristic of a numerical insensitivity to certain elements of $\bar{\eta}$ in the neighborhood of the minimum. It is believed that this insensitivity results from an ill-conditioned state in the equations which form the basis for coefficient identification. Thus, although an optimum set, say $\bar{\eta}^*$, is located and the trajectories are in excellent agreement with the experimental records, the numerical uniqueness of the elements in $\bar{\eta}^*$ remains in question due to the insignificance of specific terms in the state-variable equations.

Discussion and Conclusion

The results of this paper indicate that the dynamic system parameters can be identified using the criteria of a least squares residual function. Reasonable initial esti-

Table 2 Results of numerical optimization-coefficient identification

Coefficients	Initial set $\bar{\eta}^0$	Optimal set $\bar{\eta}^*$
$Z_{\dot{w}}'$	-0.04	-0.0666
$Z_{\dot{q}}'$	-0.0015	-0.3529
$M_{\dot{w}}'$	0.0	-0.0890
$M_{\dot{q}}'$	-0.0003	-0.3272
Z_w'	-0.03	-0.0248
Z_q'	-0.013	-0.0168
M_w'	0.009	-0.0229
M_q'	-0.01	0.0113
$Z'\delta_e$	-0.007	-0.0734
$M'\delta_e$	-0.001	-0.0684
Error ϵ	1.5×10^2	1.8×10^{-2}

Table 3 Coefficient variation for nearly equivalent trajectories (ϵ = 0.0185, ϵ^* = 0.0181) in the neighborhood of real data[a]

Hydrodynamic coefficients	Variation $\times 10^2$		Equivalent coefficients (Eq. 17)	Variation	
	ϵ	ϵ^*		ϵ	ϵ^*
$Z_{\dot{w}}'$	0.48	−6.66	$\mathbf{C}_{11}$	−2.47	−1.77
$Z_{\dot{q}}'$	−12.9	−35.3	$\mathbf{C}_{13}$	3.00	2.55
$M_{\dot{w}}'$	−0.03	−8.90	$\mathbf{C}_{31}$	0.38	0.14
$M_{\dot{q}}'$	−3.49	−32.7	$\mathbf{C}_{33}$	−0.67	−0.49
Z_w'	−0.63	−2.48	$\mathbf{D}_{11}$	1.44	1.31
Z_q'	−2.98	−1.68	$\mathbf{D}_{33}$	−3.89	−3.77
M_w'	0.09	−2.29	$\mathbf{M}_{11}$	91.3	10.3×10^3
M_q'	−0.45	1.13	$\mathbf{M}_{13}$	31.9×10^1	11.1×10^3
$Z'_{\delta e}$	−2.69	−7.34	$\mathbf{M}_{31}$	21.9×10^{-1}	31.3×10^2
$M'_{\delta e}$	−0.82	−6.84	$\mathbf{M}_{33}$	62.6	31.9×10^2

[a] The "variation" of a given coefficient is represented by the difference between typical values when ϵ = 0.0185 and $\epsilon = \epsilon^*$ = 0.0181.

mates of the parameters are not essential to guarantee convergence of the search routine. A possible explanation for the slow rates of convergence observed in the neighborhood of the minimal value of ϵ is outlined as follows.

The governing equations (4a) and (4b) may be written in the generalized form

$$M\frac{d\bar{x}}{dt} = C\bar{x} + D\delta_e + \bar{h}(\bar{x}) \tag{16}$$

where M, C, and D are ($\bar{2} \times 2$) coefficient matrices. The vector function $\bar{h}(\bar{x})$ is composed of the nonlinear terms in Eqs. (4a) and (4b). Presently, the approach is to identify the hydrodynamic coefficients which are located in the matrices M, C, and D. For a neutrally buoyant vehicle with the mass center located at the center of buoyancy, $\bar{h}(\bar{x}) = 0$, the elements of M, C, and D cannot be identified uniquely. Thus, the magnitude of $\bar{h}(\bar{x})$ may be viewed as a measure of the identifiabilty of the hydrodynamic coefficients. The $\bar{h}(\bar{x})$ function for the investigated vehicle and for many torpedo-like vehicles is small in relation to the magnitude of the other terms in Eq. (16). Thus, ill-conditionedness in the presence of small $\bar{h}(\bar{x})$ offers a possible reason for the slow convergence and floating values of the coefficients as the minimum is approached. By redefining the form of the system parameters and writing

$$\frac{d\bar{x}}{dt} = \mathbf{C}\bar{x} + \mathbf{D}\delta_e + \mathbf{M}\bar{h}(\bar{x}) \tag{17}$$

the suspected source of insensitivity is relocated in the **M** matrix while **C** and **D** possess the predominant system characteristics. The argument and procedure suggested above is substantiated by the results shown in Table 3. In the slow descent to the "minimal" error ϵ^*, the large variations observed in the hydrodynamic coefficients are reflected only in the **M** coefficients of $\bar{h}(\bar{x})$ in Eq. (17). The equivalent damping **C** and elevator **D** coefficients are apparently reliable with relatively small variations during the search near the optimum. Thus, by fixing the elements of **M** at theoretically calculated values for $Z'_{\dot{w}}$, $Z'_{\dot{q}}$, $M'_{\dot{w}}$, $M'_{\dot{q}}$, the elements of **C** and **D** may be identified effectively. With a reduced number of parameters and increased sensitivity, improved convergence in the neighborhood of $\bar{\eta}^*$ is probable. The validity of this hypothesis will be examined in a future study implementing the form of Eqs. (17) with **M** held constant.

In conclusion, a set of best-fit system parameters have been identified successively for both ideal and actual experimental data. The number of iterations required for convergence to idealized data is significantly less than the number required when using real data. At present the available analog data does not have the necessary accuracy required for an entirely realistic prediction of hydrodynamic coefficients. Thus, the refinement of the present methodology along with development of accurate digital data measurements forms the basis for future study. The present study shows feasibility and motivates continued investigation. The study of uniqueness and identifiability properties from both theoretical and numerical viewpoints will provide answers to some of the questions raised by the results obtained in this paper. It is suggested that a theoretical study similar to that presented in Ref. 6 be utilized to establish the input requirements for uniqueness and determine the bounds on the parametric errors in the present application of systems identification. Improved efficiency in the search procedure along with an established confidence in identifying the system parameters are needed refinements of the present methodology. With these refinements a very practical method of analysis will be established for immediate application to underwater vehicle dynamics. Direct extensions to generalized multiparameter, multivariable systems dynamics are certainly realizable.

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